

# On spatial resolution

## Introduction

### How is spatial resolution defined?

There are two main approaches in defining local spatial resolution. One method follows distinction criteria of point-like objects (i.e. in astronomy) and uses the image width of a point source (see figure 1), the other uses the spatial density of stripes in a “zebra print” that is imaged with a certain contrast: if the contrast gets too small the zebra print (representing some arbitrary image structure) cannot be recognized any more (see figure 2).

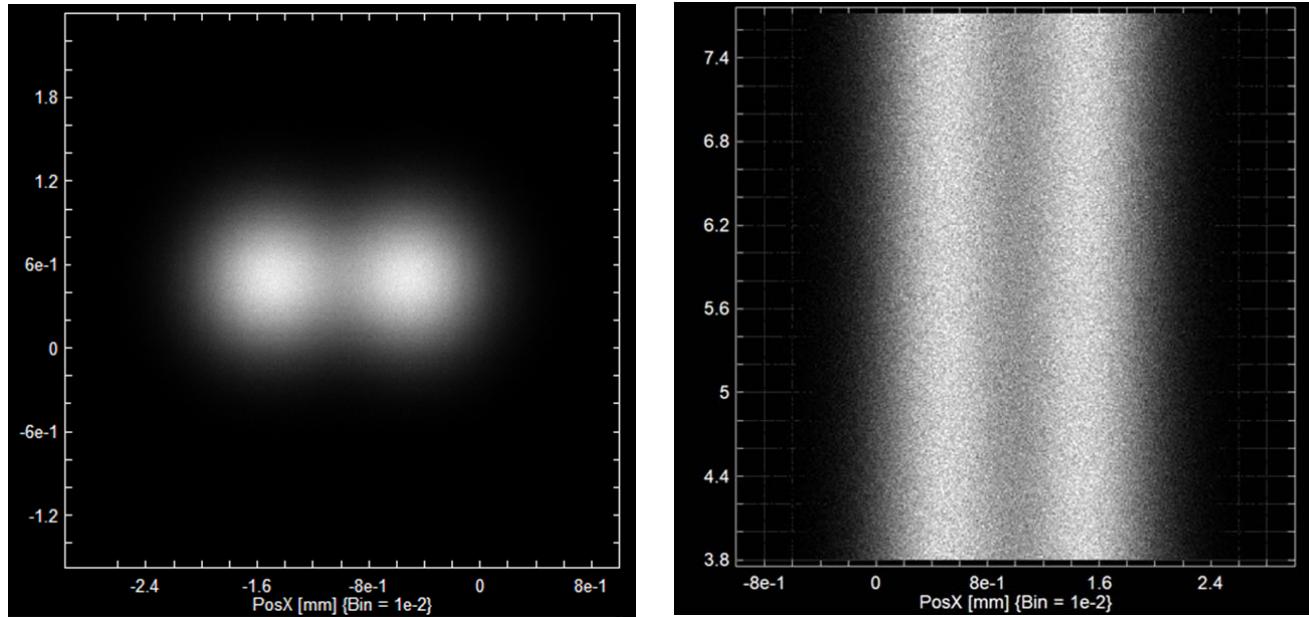
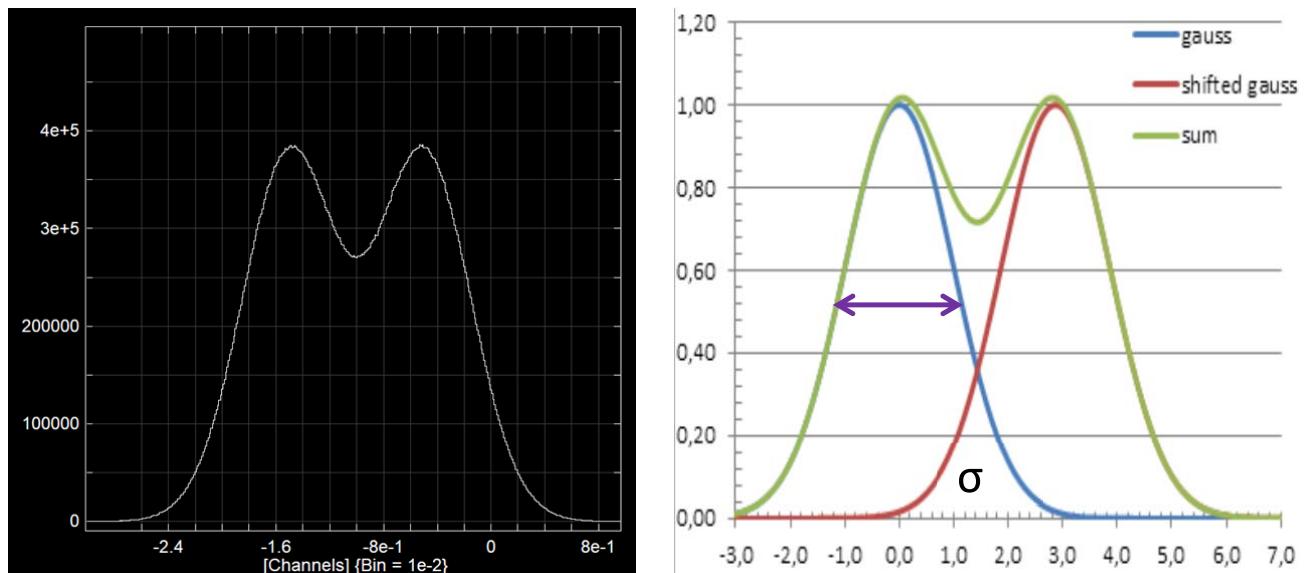
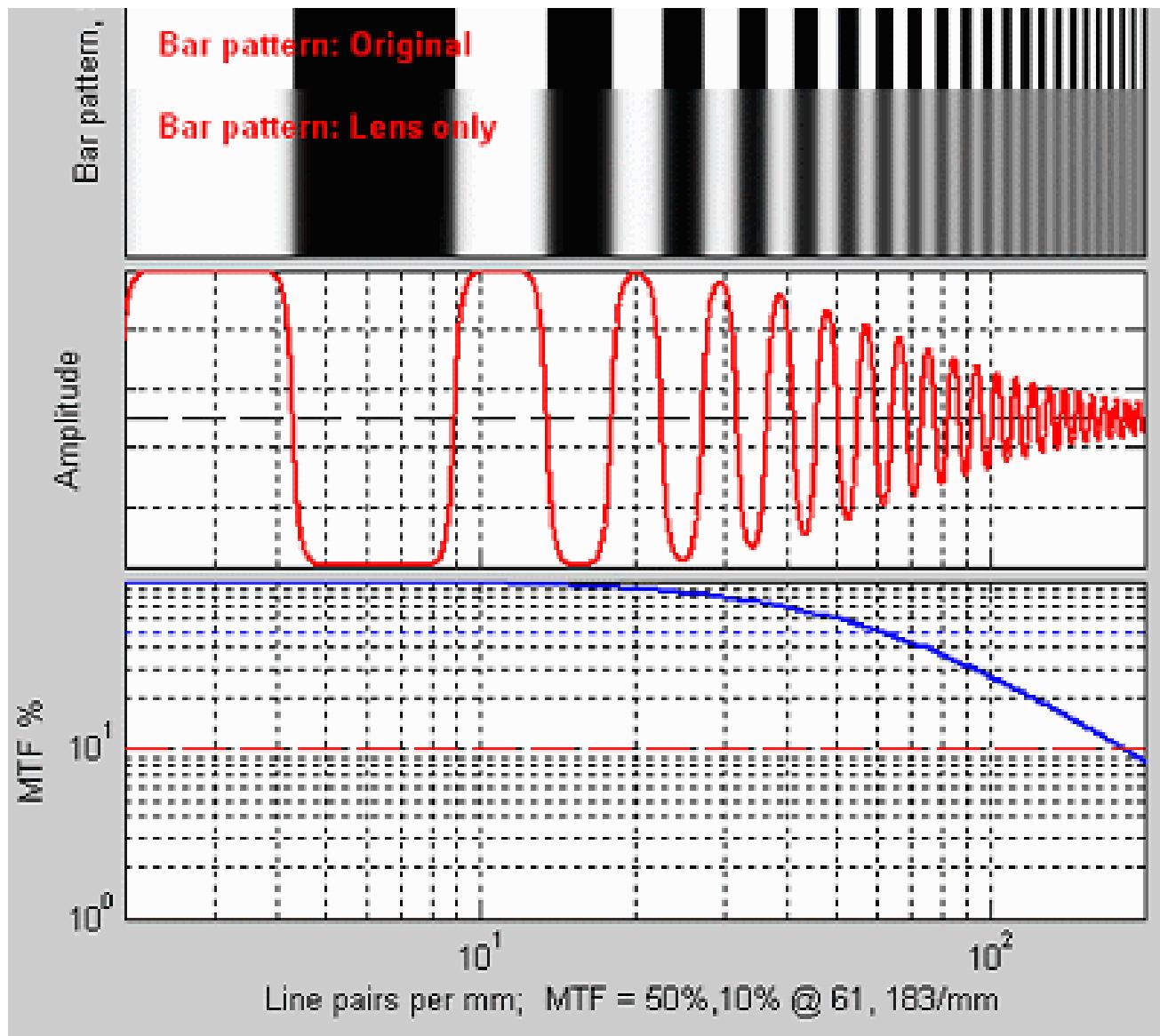


Figure 1: resolution-limited image widths of two neighbouring point-like objects and parallel narrow slits (above) at 1 mm distance and line scans through the images (projections to x axis, below left). Both images give identical projections which are assumed to have Gaussian profile and can thus be simulated by a Gauss distribution (right below). The Gauss distribution's full width at half maximum (FWHM) is  $2.35 \cdot \sigma$ . The spatial resolution in this simulation was chosen as FWHM = 0.82 mm to meet the so called Rayleigh criterion which is commonly used in astronomy: two point-like objects can be distinguished if the FWHM resolution of the optics is 0.82 times the distance of their images. It is to note that the Rayleigh criterion is only one of several criteria used (see below).



The contrast-based approach for defining resolution does not use a distance but a stripe (line) density “line pairs per mm” (lp/mm) of a zebra print (see figure 2) which results in a certain image contrast. Contrast is defined as the scaled ratio between the grey scale amplitudes  $I$  (“brightness”) in the image, where “white” has 100% intensity amplitude and “black” zero amplitude (= “dark”). The contrast between two neighbouring stripes of different intensity is defined as  $(I_{\max} - I_{\min})/(I_{\max} + I_{\min})$ .



Taken from <http://www.imatest.com/docs/sharpness/>

*Figure 2: Zebra print (bar pattern) with increasing stripe density (width) from left to right and the image response of a resolution-limited optics. Depending on resolution the grey scale amplitude will at some line density not anymore reach black/white saturation and only a certain grey scale (amplitude) contrast is maintained. The so-called mean transfer function (MTF) determines the ratio between image contrast and object contrast (the latter is by definition 100% here). The stripe (line) density (lp/mm) where this MTF contrast ratio reaches 50% serves as a common criterion for the resolution (i.e. when  $I_{\max} = 75\%$  and  $I_{\min} = 25\%$ ). Also here, other criteria can be used, such as the density at MTF ratio (contrast) 10% where the stripes are considered to become “indistinguishable”.*

A “line pair” (lp) is the combination of adjacent black and white stripes. The inverse value of lp/mm is therefore the center-to-center distance between two white stripes. Stating the resolution as line density (lp/mm) or “frequency” can be seen as a Fourier transformation of the resolution definition via FWHM width of a point image, the relation is:

$$\text{lp/mm (@MTF50\%)} \quad \text{times} \quad \text{FWHM (in mm)} = 1/2$$

### How will different resolutions affect image quality?

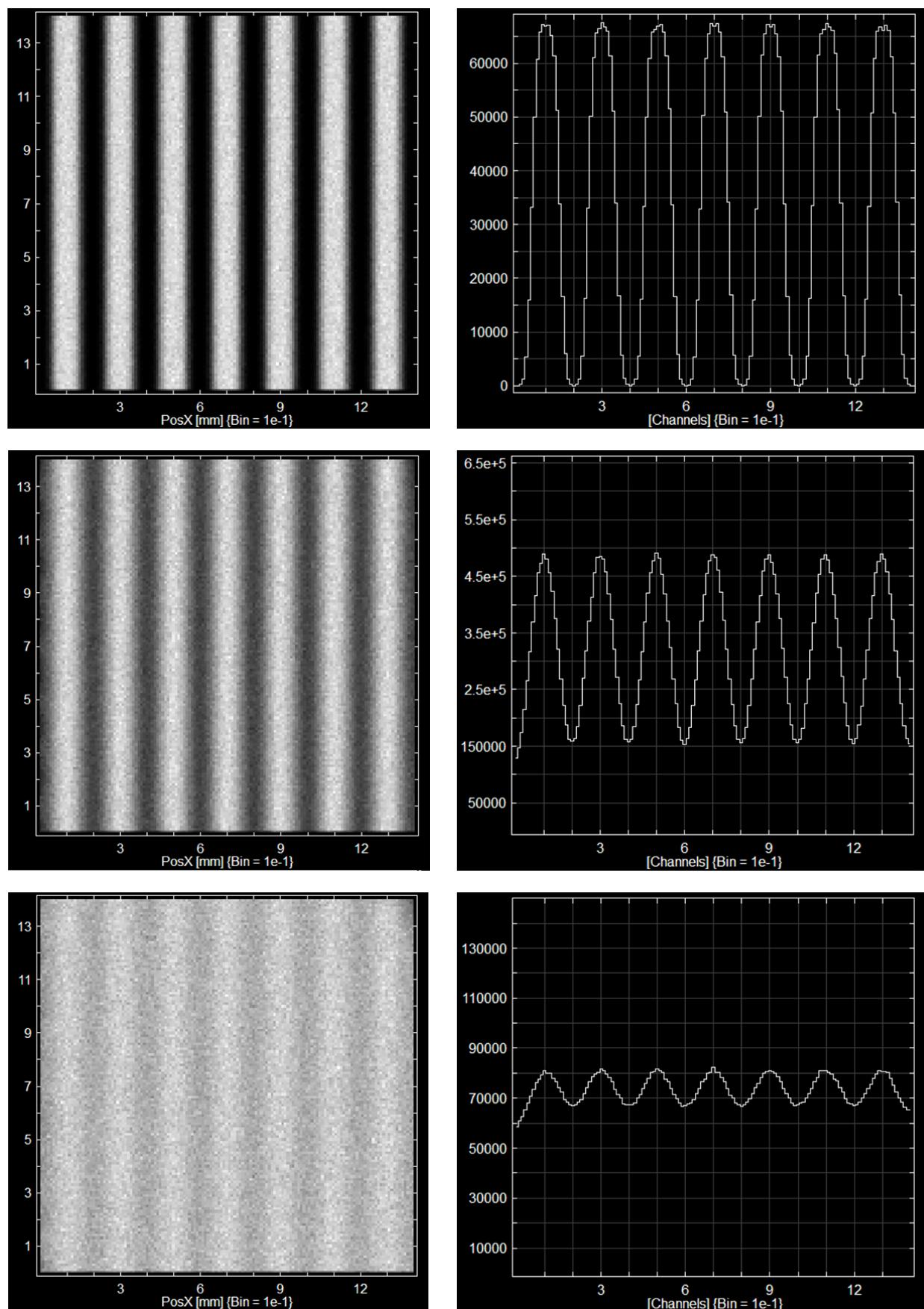


Figure 3: Image response to a zebra print with 0.5 lp/mm at different FWHM resolutions of 0.333 mm, 1 mm and 1.7 mm (from above). Contrasts are found to be near 100%, 50% and 10%, respectively.

Figure 3 (above) shows the image response to a zebra print for different resolutions. It turns out that at “MTF50” (middle pictures) the FWHM is as wide as one stripe (or half a “line pair distance”) as can also be seen in Figure 4 (red broken line).

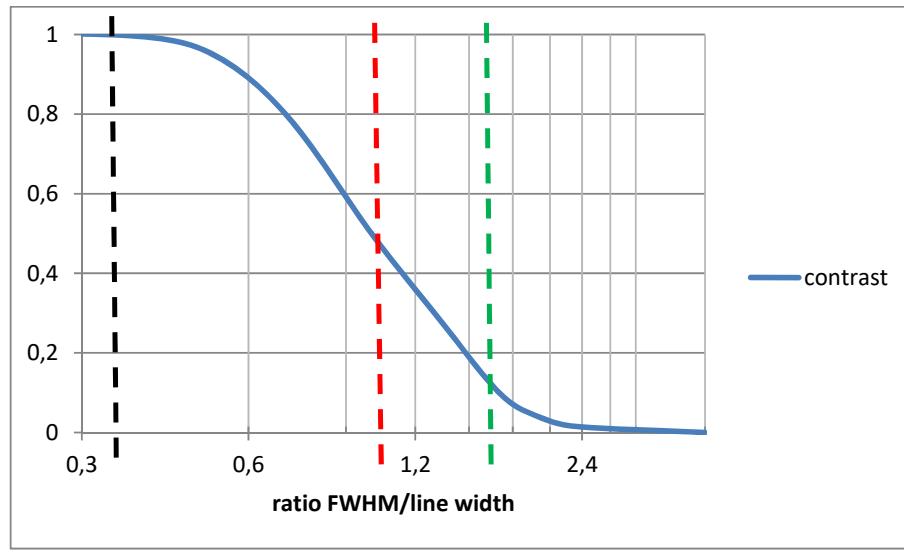


Figure 4: Image contrast as function of resolution in FWHM. Maximum image contrast is maintained until the FWHM resolution is as big as 1/3 of the stripe width (black broken line). MTF10 (10% contrast, green broken line) is reached at FWHM resolution of 1.7 stripe widths, where the zebra pattern is considered to finally disappear under practical observation conditions. The red broken line indicates MTF50 with FWHM = stripe width.

Which resolution criterion can be considered as “sufficient” depends on the application and, in case of images achieved by “counting” individual particles or photons, largely on the acquired number of counts per image pixel, i.e. the “statistics” in the image. Additional background counts will also adversely affect image quality. For point sources at good statistics the criterion whether resolution-limited discs are considered recognizable as originating from separate sources is the Sparrow criterion (see figure 5).

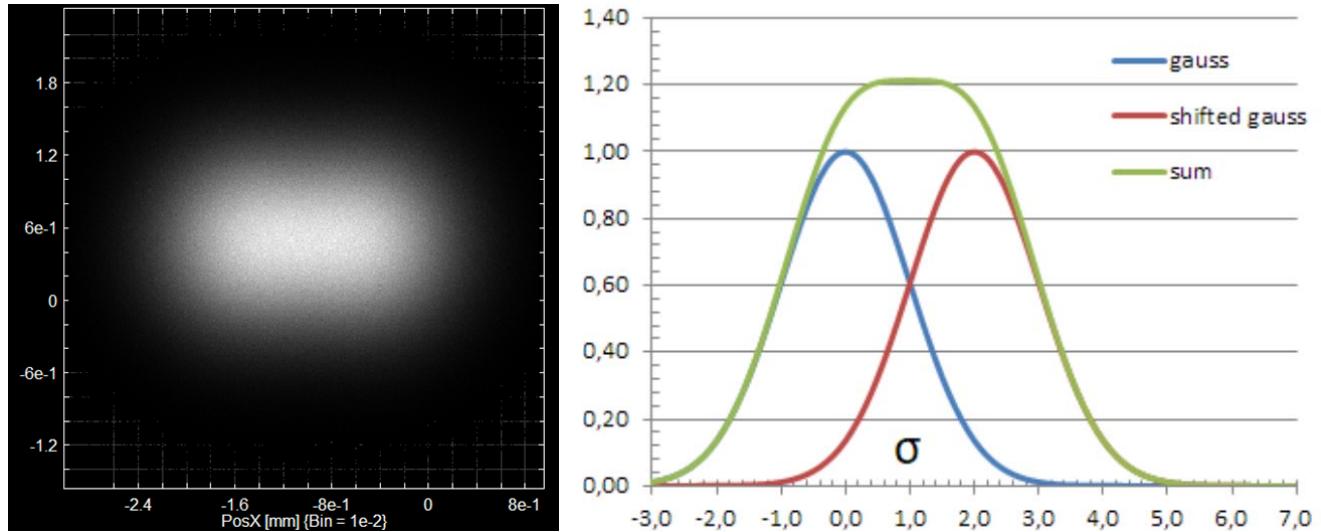
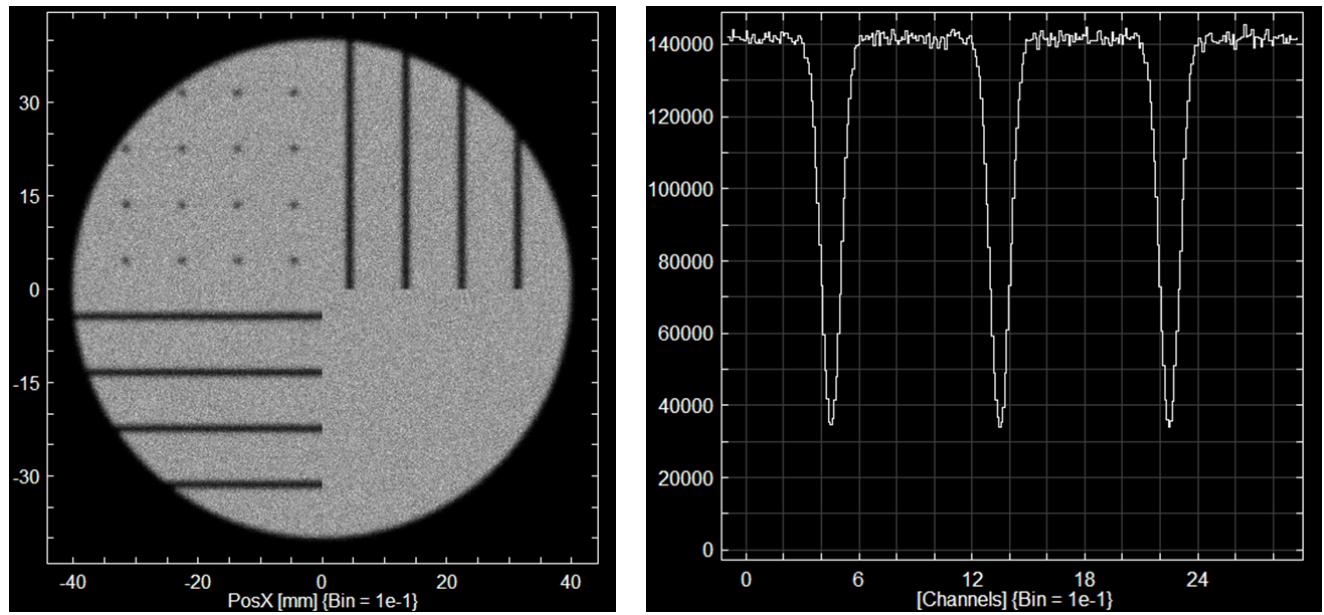


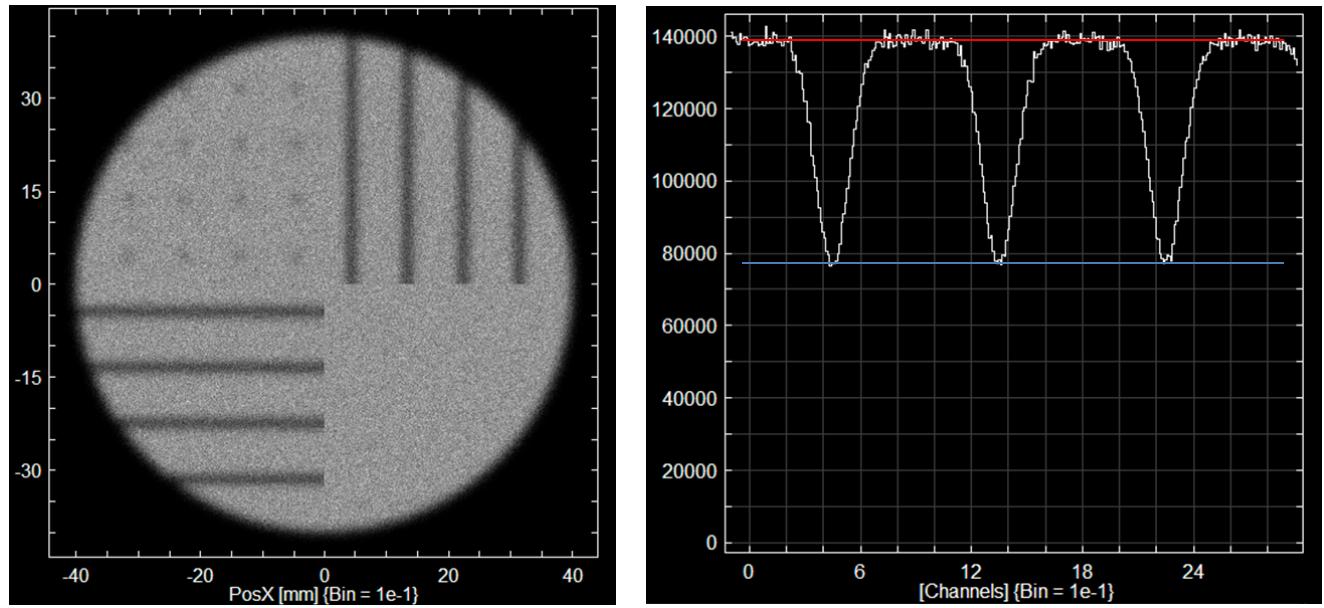
Figure 5: If two resolution-limited (“Airy”) discs, here approximated having Gaussian profiles, get as close as  $2\sigma$  (0.85 FWHM), the minimum in the line scan through the discs disappears. This is the so-called Sparrow criterion which defines whether two point sources appear distinct or not.

Also this criterion holds only for good imaging statistics, low background and if the intensity in both discs is well within same order of magnitude.

For images containing structural information in form of locally reduced intensity (“shadow”) other criteria may be considered. Figure 6 shows resolution-limited images of a “shadow mask” consisting of slits and apertures with 1 mm widths/diameters at two different resolutions.

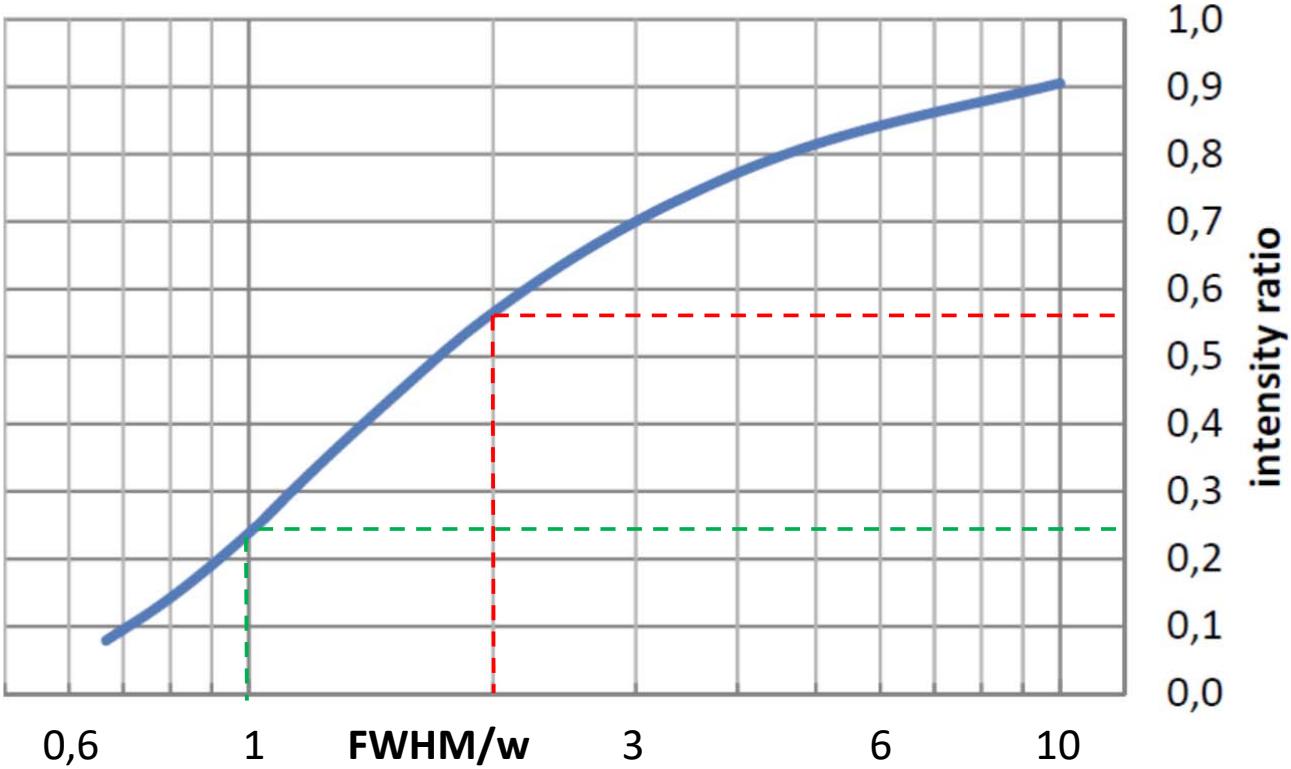


**Figure 5:** images of a “shadow mask” consisting of slits and apertures with 1 mm widths/diameters at FWHM resolution 1 mm (above) and 2 mm (below). While the slit shadows would still be well recognizable even at inferior resolution (well beyond twice the slit width) the grey scale intensity in the aperture images below merely drops to 90% of the flat field illumination intensity (in above image to about 50%).



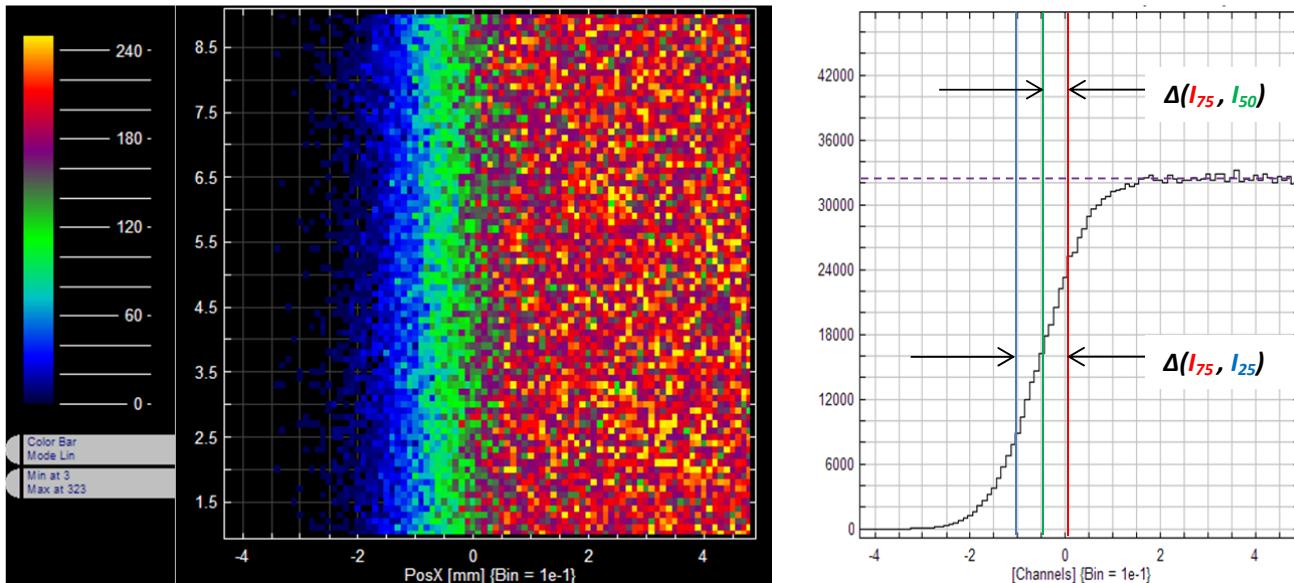
As a thumb-role one can assume that recognition of “inverse” (dark) point-like structures requires a FWHM resolution on the order of their diameters, while line structure shadows remain prominent even if their width is just on the order of  $1\sigma$  resolution ( $= \text{FWHM}/2.35$ ).

A common method to measure resolution of a detector or imaging optics is the analysis of image response when illuminating such shadow masks of distinct slit patterns (see also Fraser *et al.*, NIM A273, 1988, pp667). The ratio of the minimum intensity (blue line in Fig. 5) relative to the flat-field intensity (red line) can be used to determine FWHM resolution for a given shadowing slit width  $w$ .



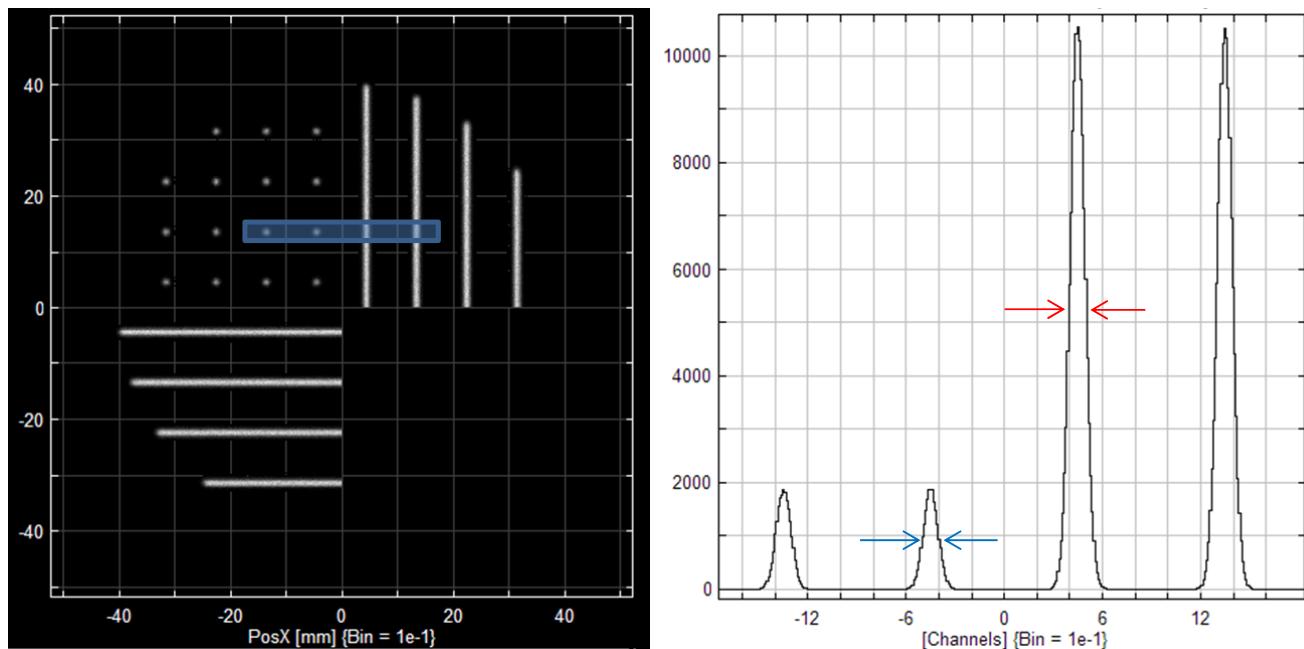
**Figure 6:** Relation between intensity ratio (as in figure 5) and FWHM for a resolution limited intensity drop caused by the shadow from a strip with width  $w$ . The dashed lines show values for simulations with FWHM = 1 mm (green) and FWHM 2 mm (red).

Also the blur of a white-to-black “knife-edge” shadow can be used to determine spatial resolution. The intensity drop profile from the plateau towards the shadow edge (see figure 7) is a function of spatial resolution and can be determined by assessing the positions where the intensity has dropped to 75% (red line), 50% green line and 25% (blue line) of the plateau value (horizontal line).



**Figure 7:** Transition profile of intensity (colour coded for better visibility) of a knife-edge shadow blurred by imaging resolution (here: FWHM = 2 mm). The vertical lines in the image of the intensity profile (right) indicate the position of certain relative intensity lines  $I_{75}$ ,  $I_{50}$  and  $I_{25}$ . It turns out that the resolution value can be determined by the distance of these intensity lines, for example:  $\Delta(I_{75}, I_{25}) \cdot \sqrt{3} = \text{FWHM}$  or  $\Delta(I_{75}, I_{50}) = 0.3 \cdot \text{FWHM}$ .

Another (more direct) method is to place an “inversed” shadow mask over an imaging detector, or project a narrow intensity profile with optics onto it. This may have the form of tiny spots (pinhole mask) or narrow lines (slit mask). Figure 8 shows the image response of such a “white” mask with hole/slit width 0.5 mm at 1 mm FWHM resolution.



**Figure 8: Illumination through a mask of tiny pinholes (0.5 mm diameter) and horizontal/vertical slits with 0.5 mm widths. The corresponding line profiles (projected only from the grey area in left picture) allow determining the resolution (here: 1 mm FWHM).**

The intensity profile will directly show the resolution characteristics (i.e. FWHM) if the aperture size or line width is small compared to the measured FWHM.

As the use of tiny aperture/slit sizes comes with low transmission one has to make a compromise if the illumination intensity is limited. Allowing larger mask width brings about resolution unrelated broadening of the intensity profiles which has to be considered when assessing resolution from measured line widths. Figure 9 shows the increase of the effective line width for different hole/slit sizes at a given resolution. Note, that the effective line width cannot simply be modelled using the law of error propagation for weighing real FWHM with aperture/slit size.

Care must be taken in the placement of the mask in proximity with the detector, especially if the illuminating source is not point-like. For extended sources edge-blur (penumbra with size  $\Delta w^*$ ) becomes significant and contributes to additional image broadening not related to resolution or finite-size aperture effects. The extra broadening due to penumbra effects for  $\Delta w/\text{FWHM} = 1$  in combination with different hole size is indicated by the arrows in figure 9. To control this effect  $\Delta w$  must be kept small relative to the measured FWHM resolution (by increasing source distance and/or reducing the mask distance to the detector), ideally to below  $\Delta w/\text{FWHM} = 0.3$  where the broadening from penumbra presence is reduced to about 2% of measured FWHM. As increasing the source distance or shading an extended source in order to reduce the effective size will reduce intensity it is usually preferable to mount a mask directly in front of the detector.

Proper mask placement, ideally in contact with the focus plane of a detector is also important if any other method of resolution measurement is employed, e.g. those proposed in figures 6 and 7.

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\*  $\Delta w = S \cdot d/D$  with  $S$  = source extension,  $d$  = distance sensor to mask and  $D$  = distance to source ( $d \ll D$ ). Example: a 10 mm diameter source at 10 cm distance from a sensor causes a penumbra extension  $\Delta w = 0.3$  mm if the mask is placed at 3 mm distance from the sensor.

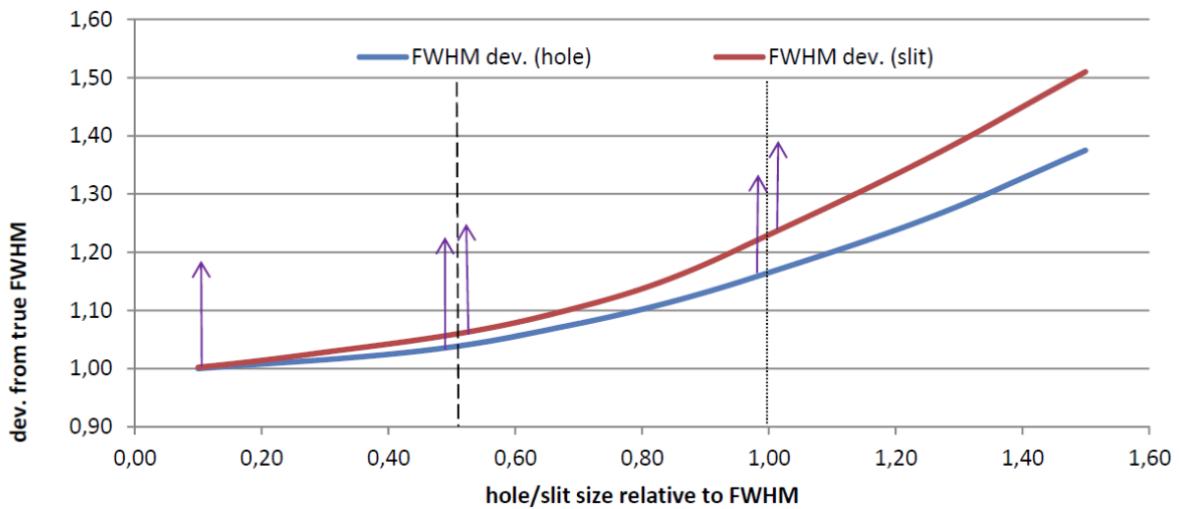


Figure 9: relative increase of the line width size if the aperture size/slit width of a mask pattern as in figure 8 is not minimized. The relative increase from the resolution-limited line width (here: 1 mm) is different for aperture and slit shape. It turns out that the size effect can be neglected if the hole size is  $< \frac{1}{2}$  FWHM (as chosen in figure 8, dashed line) but becomes already remarkable once the size reaches the FWHM value (dotted line) and must then be taken into account. The arrows indicate the amount of additional width increase caused by penumbra effects (explanation see text).

#### Sensor granularity limiting detector resolution

The finite resolution of a detector may be caused by different independent limitations of its imaging components. For example in case of a camera there is not only the lens optics to consider but also the sensor, nowadays typically a pixel device (e.g. CCD). Any digital imaging detector (and especially counting detectors) will at some point sort the image information into bins of a two-dimensional histogram. Obviously, a finite bin size will eventually limit the resolution. Ideally, this bin size should be chosen small compared to the resolution limitation given by the other components of the imaging system. Due to technical/practical limitations this is not always achievable so that bin (pixel) size effects on the overall resolution usually must be taken into account.

When determining resolution via investigations of the MTF50 formalism for CCD cameras it turns out that the pixel size must be smaller than half a line pair (so called Nyquist-frequency =  $0.5/\text{pixel-size}$ ). This means that the FWHM resolution of a digital imaging detector cannot be better than twice the pixel or bin size. Figure 10 shows in more detail the increase of line width for a given FWHM resolution as function of bin size.

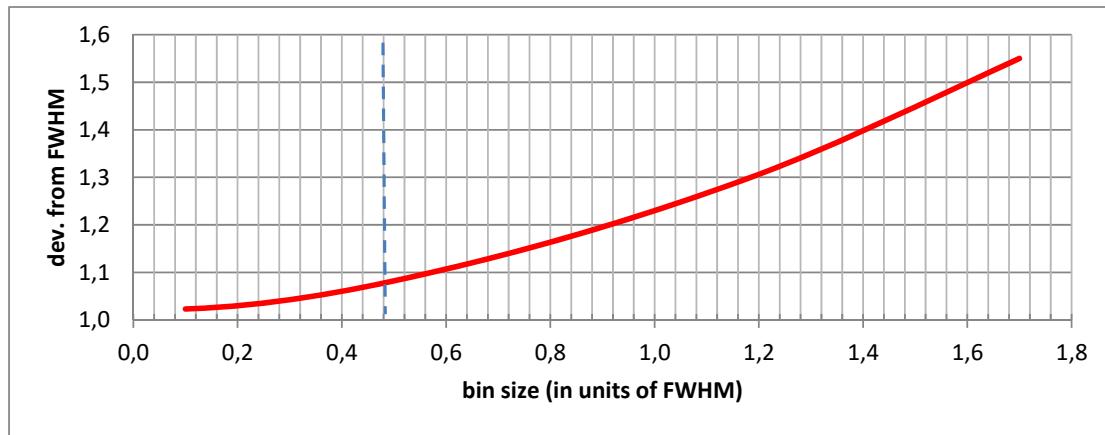
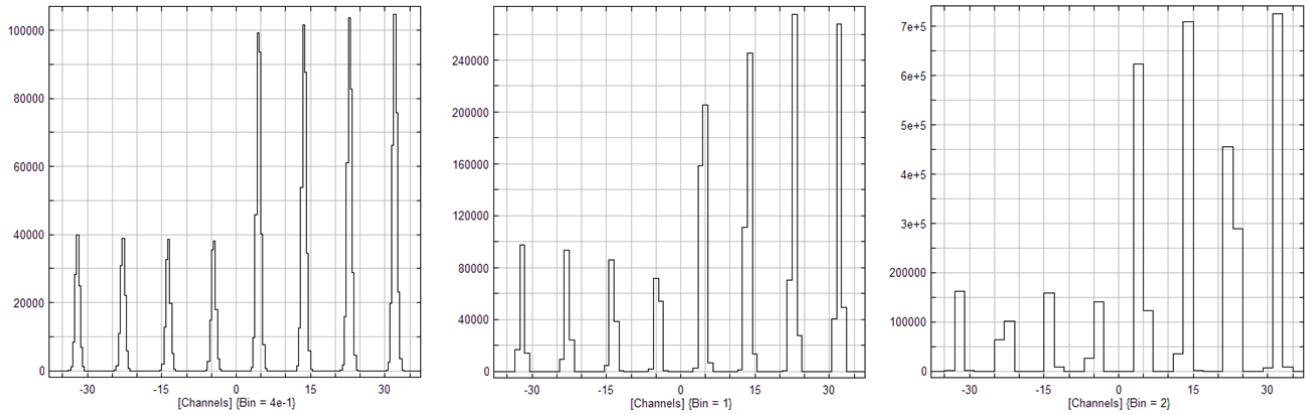


Figure 10: measured line widths as in figure 8, right picture for different channel bin sizes (see also figure 11). The “true” FWHM deteriorates remarkably as the bin size gets larger than half of the true FWHM, see dashed line. This is in accordance with the Nyquist theorem.

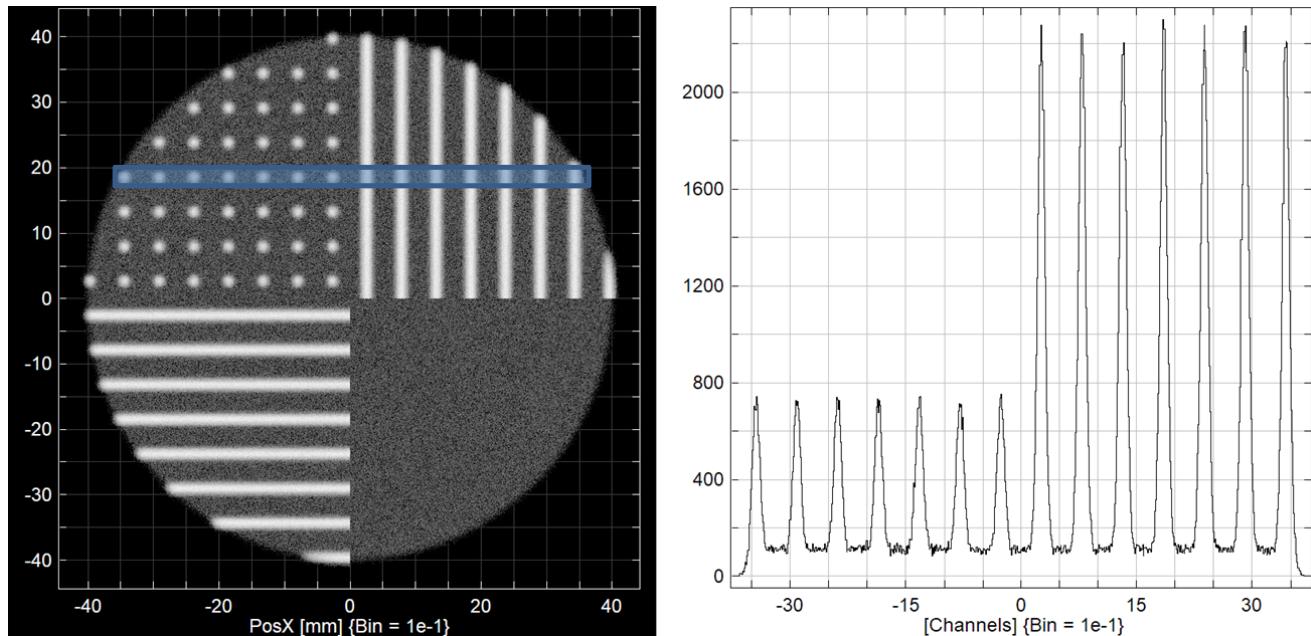
In some cases the available bin size may be the only or the main limiting component of a detector. Once the bin size gets larger than the FWHM resolution limit due to other detector components, structures of smaller-sized object (like narrow lines) will not be represented properly anymore as can be seen in Figure 11.



*Figure 11: line shapes as from figure 8 after re-binning to channel widths of 0.4, 1 and 2 in units of imaging FWHM (from left to right). If the bin size is as large or bigger than the “true” FWHM footprint of a small structure the shape information gets lost as can be seen in the middle and right picture. A structure is only fairly represented if the bin size is smaller than half of the true FWHM value (left picture), which is in accordance with the Nyquist theorem.*

#### Limitations imposed by poor statistics (low illumination)

In case of poor imaging statistics (“low light level”) re-binning of data may improve an image because it increases the number of counts in each pixel. This is especially beneficial in presence of background counts. Figure 12 shows the detector response to a test mask limited by 1 mm FWHM resolution. The mask is semi-transparent (7%) which yields some background intensity. The same images at low statistics are presented in figure 13. The comparison shows that maximizing spatial resolution is not the only criterion for achieving optimal imaging data when low intensity introduces statistical noise. Such a situation is typically reached when the number of counts per bin gets below 100, introducing 10% average variation ( $\sigma$ ) between neighboring channels that have been uniformly illuminated.



*Figure 12: Detector response to a semi-transparent test mask (allowing background counts) at high image statistics. The 2d plot in left image uses logarithmic scale, the right picture shows the projection (line scan) from the marked region (in linear scale).*

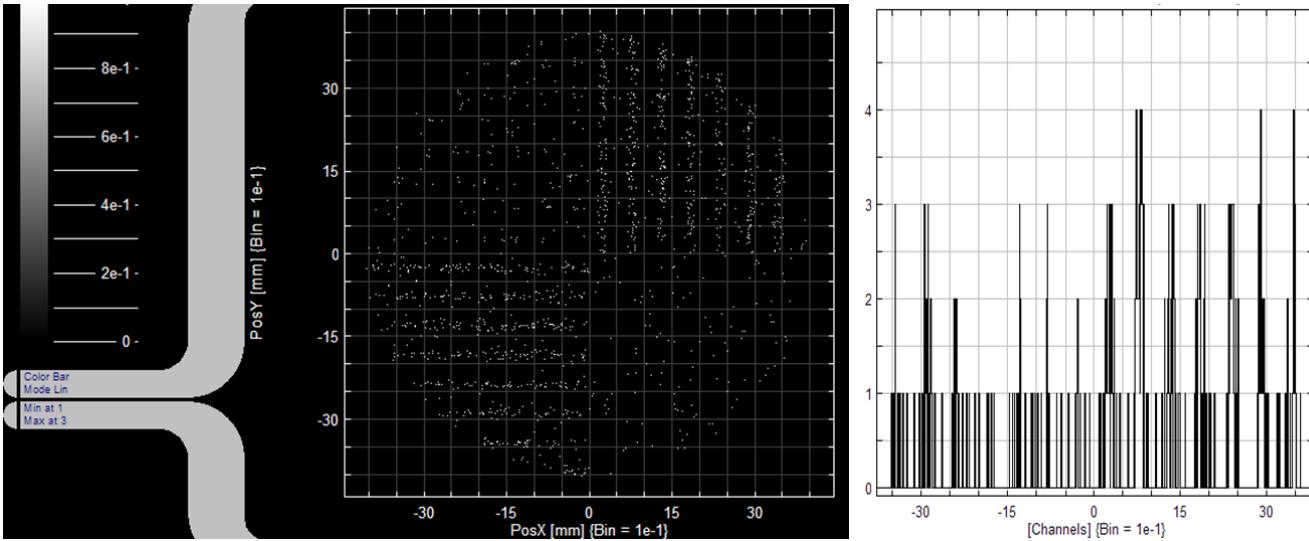
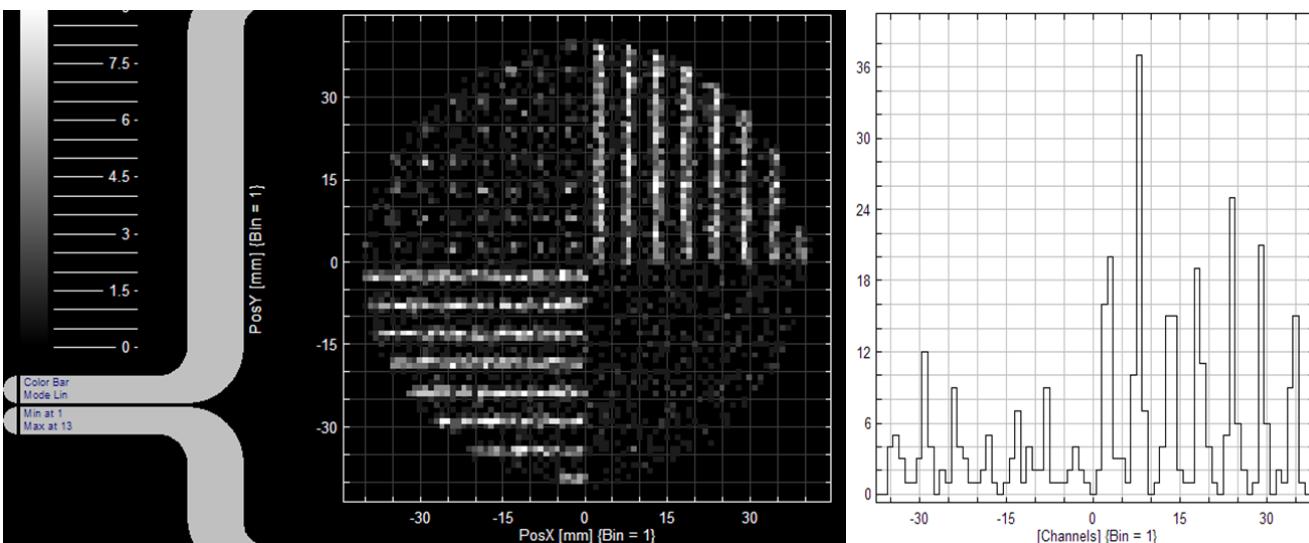
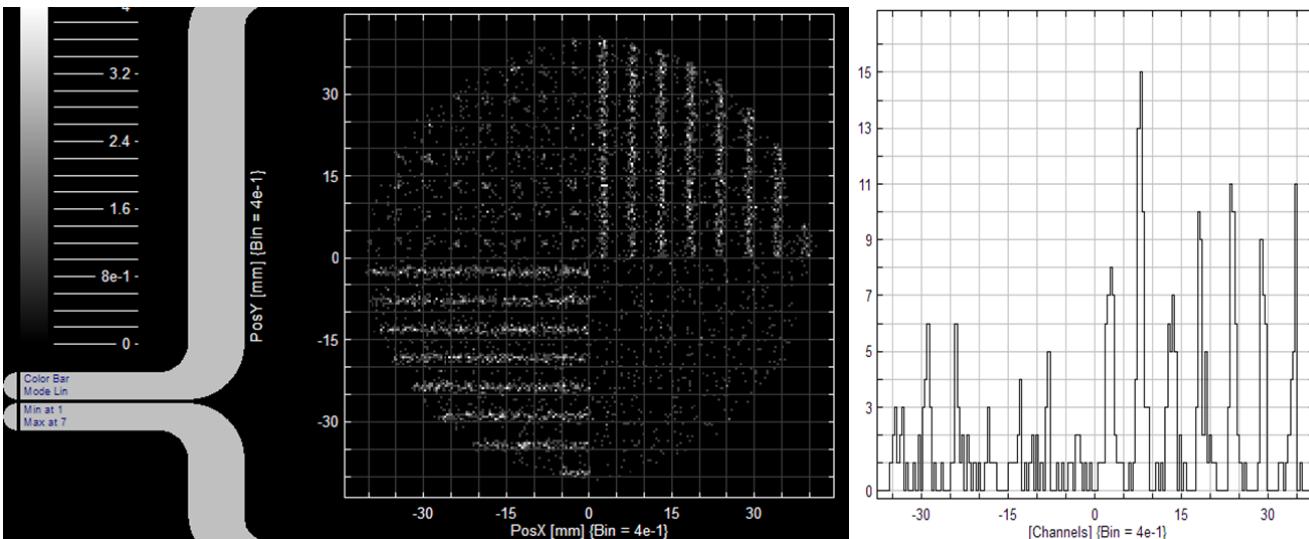


Figure 13: Above: pictures as in figure 12 but with low statistics (low exposure time and/or low intensity). As the image content is re-binned towards bigger pixel sizes (0.4 mm in middle pictures below and 1 mm at bottom) the structures become recognizable. At 0.4 mm pixel size the Nyquist criterion is still fulfilled, i.e. resolution is maintained while at 1 mm bin size resolution is inferior (as expected from figure 10) but the statistical relevance of data is further improved. Compromise between resolution and statistical relevance must be found depending on the imaged object.

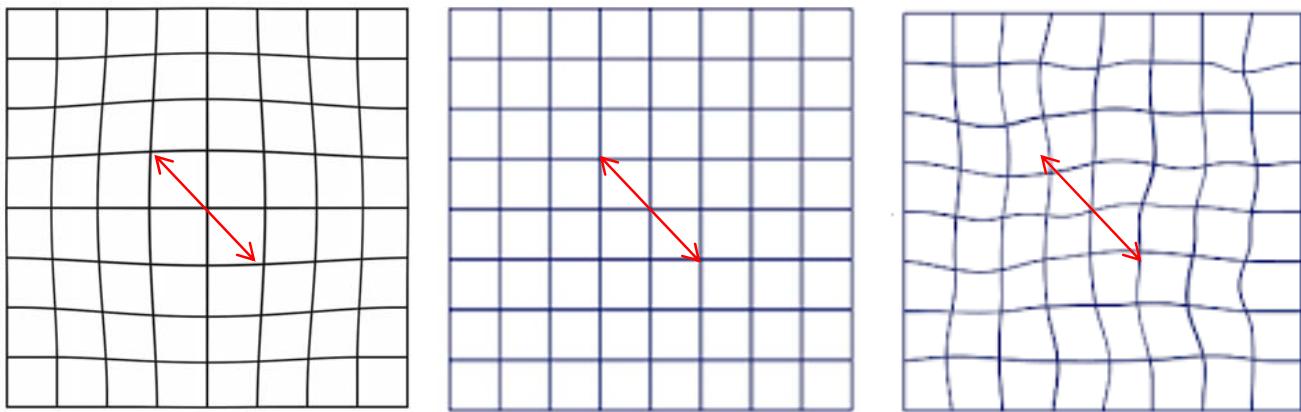


If low image statistics calls for considerable increase of bin size to achieve statistical relevance one can reach a situation where the experimentally available resolution could possibly be traded against higher intensity (if correlated) without affecting the image quality at such increased pixel size (i.e. still observing the Nyquist criterion according to the increased effective pixel size).

### Global resolution (linearity)

The term “spatial resolution” is typically considered to denote a “local” resolution which specifies the minimum size of structures that can be distinguished. This local resolution is usually independent from other imaging defects like distortion. Image distortions are characterized as deviations from linear scale correspondence between object structures and their image. Perfectly linear imaging will for example produce a distortion-free image of a straight grid pattern (see figure 14, middle picture). Typical image distortions are “pillow-” or “barrel-” deformations caused by non-perfect optics (left picture in figure 14). Even if a distortion pattern is irregular, local resolution is usually maintained although apparent position deviations may be large: the grid lines appear equally sharp but the grid structure appears “crumpled” on a larger scale.

Image distortion may be characterized by the variations of distances between grid knots which should have the same distance for perfect imaging.



*Figure 14: images of grid lines with same resolution but different linearity: barrel distortion (left) and irregular (intrinsic) distortion. Grid knots can have varying distances in spite of good local resolution.*

If an experimental task calls for measuring distances of remote points in an image, the important property of an imaging device is linearity (“global resolution”) and not necessarily the (local) resolution. The local resolution is only the limit for linearity in case of perfectly “straight” imaging properties.

### Discretisation

Pixelated imaging read-out structures reach spatial resolution much smaller than the pixel size if an extended “footprint” of charge (or light) intensity is imposed on them: by measuring the relative portions of collected intensity on each pixel (or strip) the “center-of-gravity” of the footprint can be determined with great precision (see figure 15).

For optimal resolution the ratio of strip distance and footprint size must be well-chosen. Ideally, there is a linear correspondence between the footprint center position and the value measured by weighing the relative portion projected onto the strips at any position (homogenous imaging).

If the footprint size is not perfectly controlled and can be below the ideal level, small linearity deviation occurs which scales with the distance from a strip center, such having a periodicity of the center-to-center strip distance. Although being very small, this position shift can produce remarkable intensity undulation in flat-field illuminated parts of an image acquired with such discrete-pattern read-out.

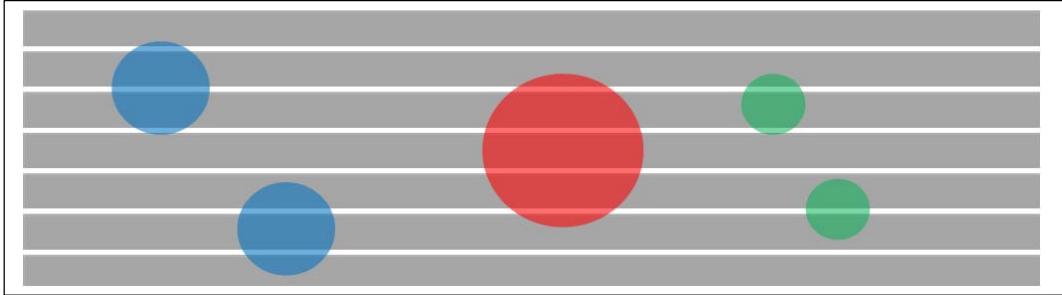


Figure 15: discrete strip anode pattern that can determine the vertical center position of “footprint discs” by measuring the relative portion of geometric exposure. The relation is linear as long as the footprint size is large enough compared to strip width/distance. This is not fulfilled if the footprint size becomes comparable with strip distance (green circles).

Figure 16 shows an image with such an undulating non-linearity (“discretisation”). The strip distance is 1 mm and the maximum periodic error in position determination is only 0.01 mm, i.e. well below strip width, spatial resolution or bin size. Nevertheless, this tiny effect produces an intensity undulation of about +/- 6% with period of 1 mm. It appears rather surprising that this small sideway shift can produce such a lateral “wave” amplitude but since this is not a random error but a collective shift of all detected events towards the same direction it can cause a remarkable effect.

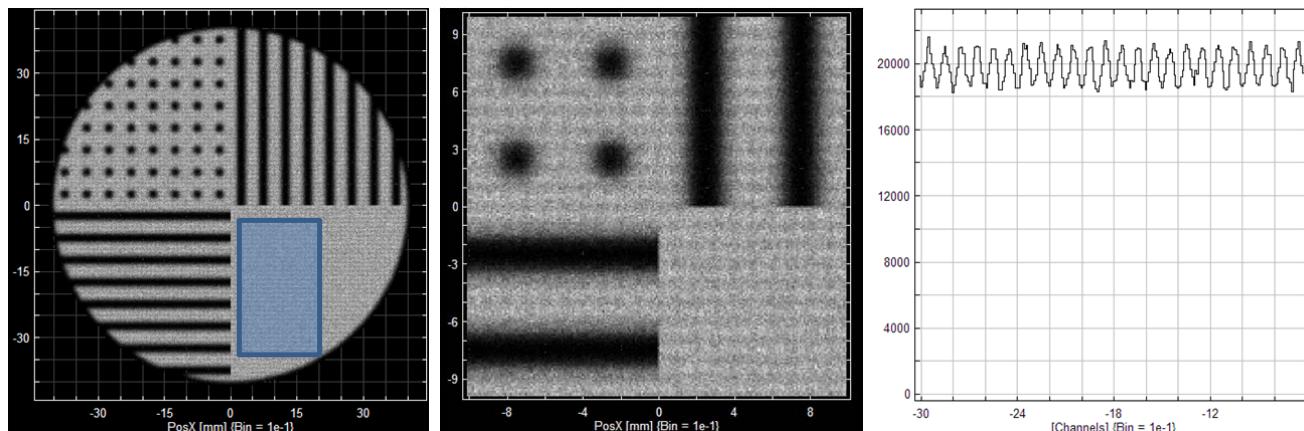


Figure 16: Simulated imaging response for systematic undulating position shift (here: up to 10 micron) with a period of 1 mm as it is sometimes observed for center-of-gravity imaging by discrete read-out patterns with this distance. The undulation causes a “discretisation” effect of the flat-field intensity, visible at high statistics. It appears prominently in a projection to one axis (here: to y-axis, data from the marked region). The resulting amplitude of intensity fluctuation is about +/- 6% of mean intensity.

Figure 17 (below) shows the undulation height of flat-field intensity caused by a certain periodic position shift (systematic position error).

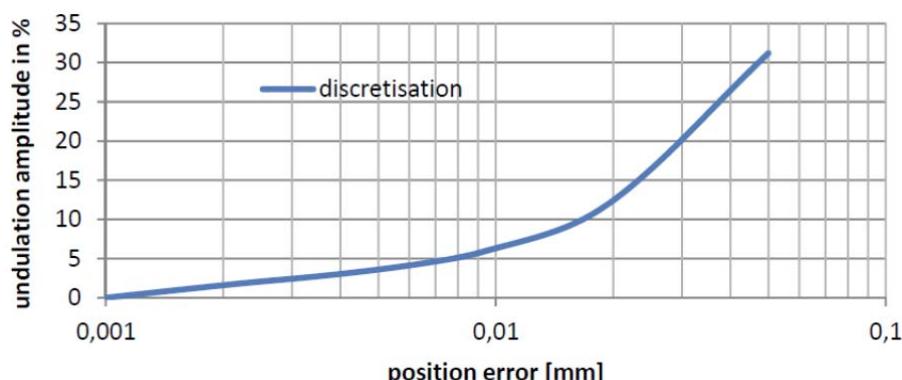


Figure 17

Such undulation in an image will obscure underlying structures with low contrast. In single particle/photon counting applications the effect on the positioning of an individual detection event can often be neglected because it is usually much smaller than the spatial resolution obtainable with pixelated read-out structures.